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LETTER TO THE EDITOR

Infrared singularities at exceptional external momenta

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Abstract. The analytic structure of the Feynman amplitude for arbitrary Euclidean external momenta is discussed.

In an elegant paper Speer (1975) investigated the structure of the analytically regularised Feynman amplitude for non-exceptional Euclidean momenta. He found the location of all possible poles of amplitude regularised with both the λ parameters of analytic renormalisation and a complex space–time dimension ν and proved the existence of the non-empty region in parameter space in which the integral defining the amplitude is absolutely convergent.

In this Letter we wish to point out that Speer's procedure can be extended to the case of arbitrary Euclidean external momenta. We have omitted proofs since they involve only slight modifications of those given by Speer. Let G be a fixed 2-connected Feynman graph with the set of N lines Ω, massive lines $\Omega^M \subset \Omega, n$ vertices θ and external vertices $\theta^E \subset \theta$. (The vertex is called external if and only if the incoming external momentum is different from zero.) We assume that either $|\Omega^M| \geq 1$ or $|\theta^E| \geq 2$. For every subgraph $H \subset G, \Omega_H$ denotes the lines of H, θ_H the vertices of H, $n(H) = |\theta_H|$ and $c(H)$ is the number of connected components of H. H is irreducible if it is 2-connected or consists of a single line. Maximal irreducible subgraphs of H we call the pieces of H. G^∞ is the graph obtained by adjoining an additional vertex v_∞ to θ connected by one line to each vertex of θ^E. If $S \subset G$ has connected components $S_1, \dots, S_k$ the graph obtained by collapsing each S_i to a single vertex we denote by G/S; the natural mapping $G \to G/S$ we denote by $P_{G/S}$.

If $H \subset G$ let G^∞/H have pieces $Q_1, \dots, Q_k$ numbered so that $v_\infty \notin \theta_{Q_i}$ and $\Omega^M \cap \Omega_{Q_i} = \phi$ for $i \geq j$. Then

$$\bar{H} = H \cup P^{-1}_{G^\infty/H}(Q_j \cup \dots \cup Q_k)$$

is called the saturation of H. Let now $X = \{H \subset G | H = \bar{H}, H \text{ irreducible}\}$. The above definitions are taken unchanged from Speer's work.

The subgraph $S\sqrt{G}$ is a link if (a) $\Omega^M \subset \Omega_S$ and $\theta^E \subset \theta_s$, (b) the sum of the external momenta coming into each connected component of S equals zero, (c) the removal of any piece of S destroys property (a) or (b).

Note that in the case when the external momenta are non-exceptional the above definition is equivalent to that given by Speer. In the general case, however, the equality $S = G$ may not be fulfilled. Further, let $Y = \{Q = G/S | \; S$ a link and Q

irreducible}. To every line $l \in \Omega_G$ we assign the complex number λ_l and define for $K \in X \cup Y$

$$\pi_K = \sum_{l \in \Omega_K} \left(\frac{\nu}{2} - \lambda_l - 1\right) - [n(K) - c(K)]\frac{\nu}{2}.$$

Then we have the following theorem:

Theorem. Let G be a 2-connected Feynman graph. For every fixed set of Euclidean external momenta the integral defining the Feynman amplitude corresponding to G converges in the region

$$Z = \{(\nu, \lambda) \in \mathbb{C}^{N+1} | \pi_H < 0, H \in X, \pi_Q > 0, Q \in Y\}.$$

Moreover it has a meromorphic extension to all of $\mathbb{C}^{N+1}$ with possible simple poles on the variates

$$\pi_H = 0, 1, 2, \ldots, \qquad \pi_Q = 0, -1, -2, \ldots$$

and for every $K \in X \cup Y$ the pole on the variety $\pi_K = 0$ is actually present. For two sets of external momenta having the same vanishing partial sums the analytic structures of the amplitudes are identical.

References

Speer E R 1975 *Ann. Inst. Henri Poincaré* **XXIII** 1